
Three layer problem - Eric Gallimore

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Setup

```
function threelayer()

close all;

theta = 0:0.01:pi/2;

c(1) = 1500; %m/s
c(2) = 1550;
c(3) = 1600;

% only the ratio matters
rho(1) = 1;
rho(2) = 1.2;
rho(3) = 1.5;

theta_in_deg = .1:1:90;
theta_in = theta_in_deg*pi/180;
d = .1:.1:10; % ratio of depth to wavelength

for i=1:length(theta_in)
    for j = 1:length(d)
        [R1, T2, R2, T3] = solve_layers(d(j), theta_in(i), c, rho);

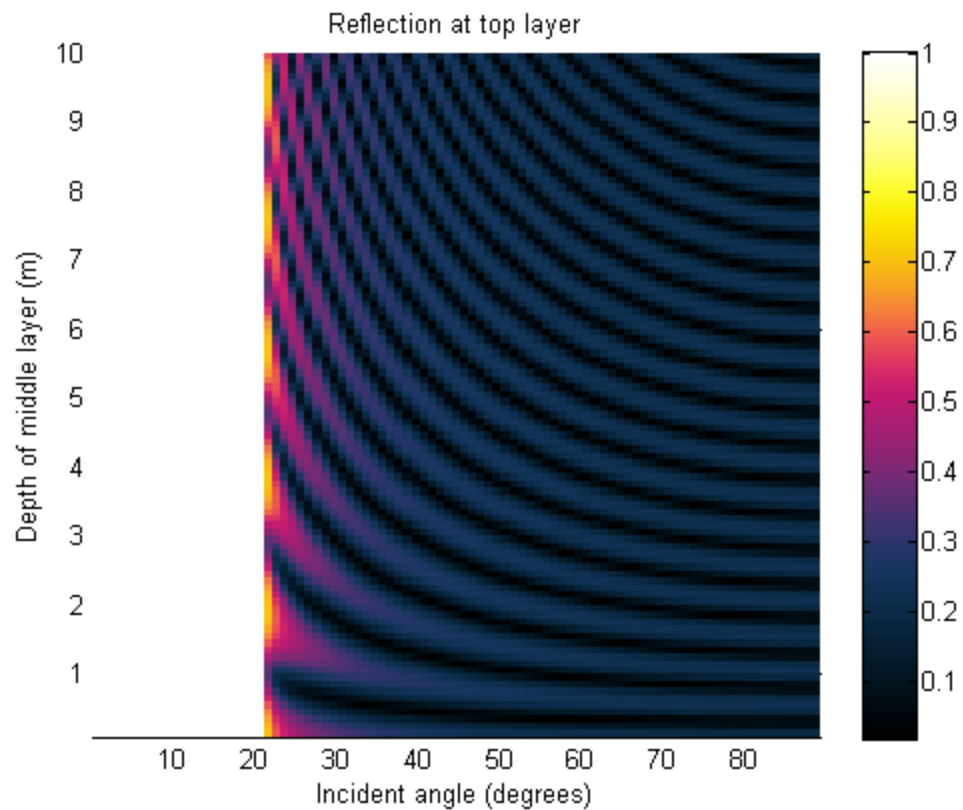
        reflect(i, j) = R1;
        transmit(i, j) = T3;
    end
end
```

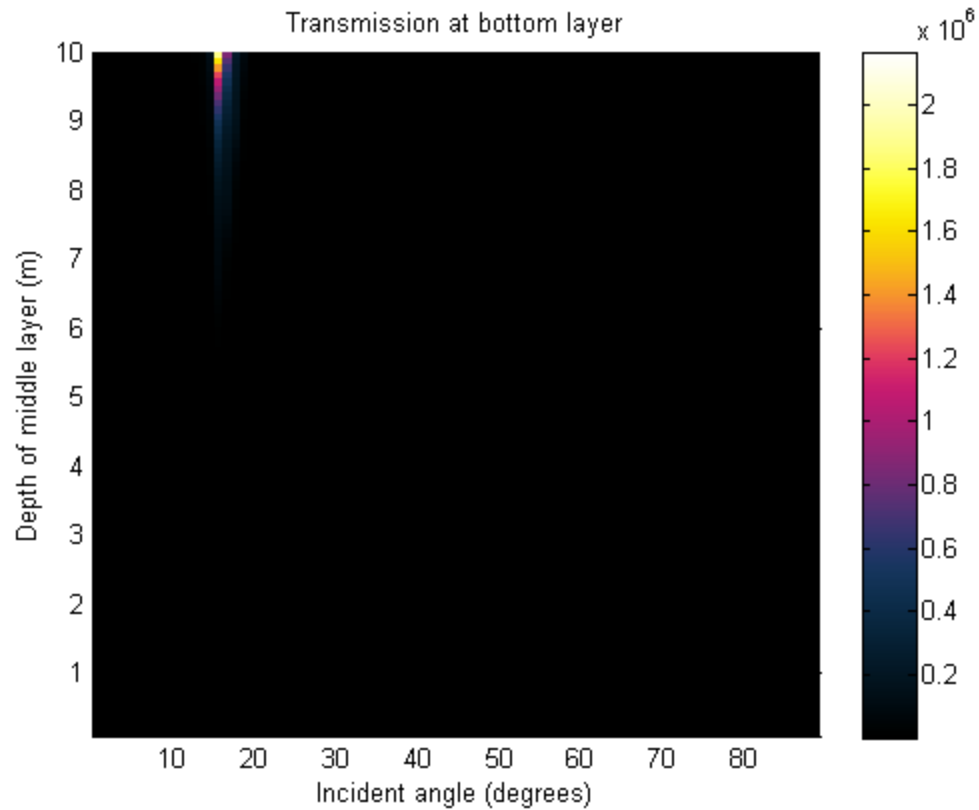
No attenuation

```
figure('name', 'Reflection at top layer');
surf(theta_in_deg, d, reflect', 'EdgeColor', 'none');
colormap(morgensstemning());
colorbar();
view([0 90]);
```

```
axis tight;
xlabel('Incident angle (degrees)');
ylabel('Depth of middle layer (m)');
title(get(gcf(), 'name'));

figure('name', 'Transmission at bottom layer');
surf(theta_in_deg, d, transmit', 'EdgeColor', 'none');
colormap(morgensstemning());
colorbar();
view([0 90]);
axis tight;
xlabel('Incident angle (degrees)');
ylabel('Depth of middle layer (m)');
title(get(gcf(), 'name'));
```





With attenuation

```
% subtract 0.5 dB/lambda attenuation
a_lambda = 0.5;
sigma = a_lambda/54.58;
c_i = -1j*sigma*c;
c = c+c_i;

for i=1:length(theta_in)
    for j = 1:length(d)
        [R1, T2, R2, T3] = solve_layers(d(j), theta_in(i), c, rho);

        reflect(i, j) = R1;
        transmit(i, j) = T3;
    end
end

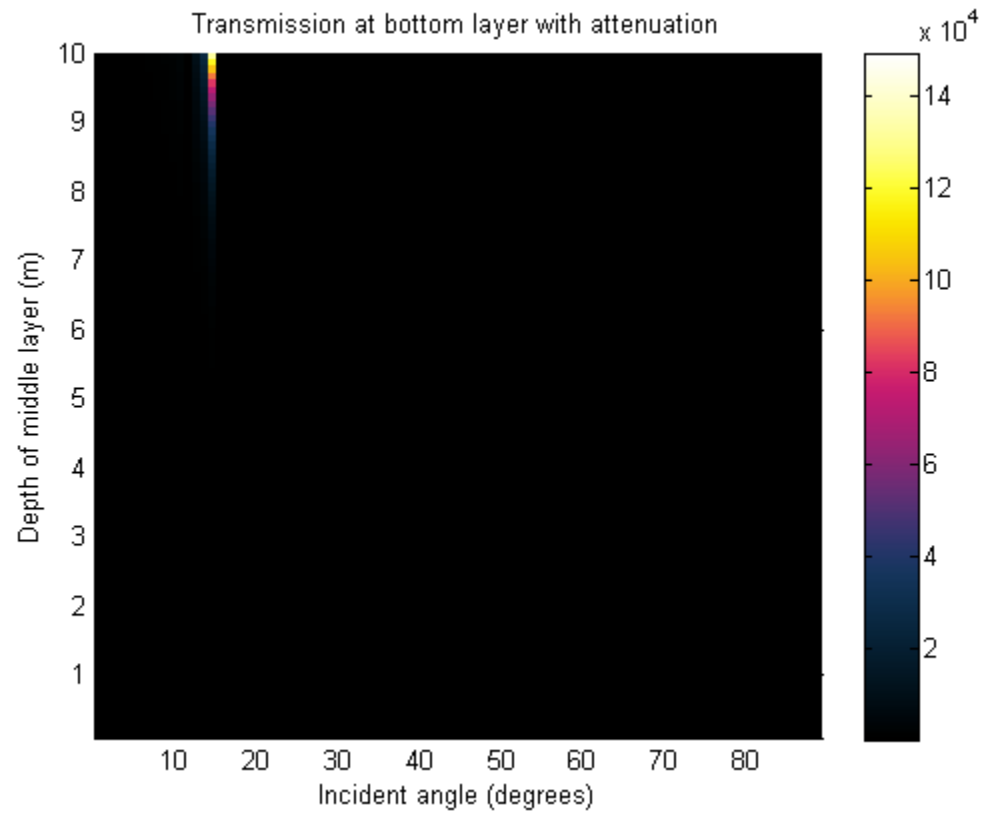
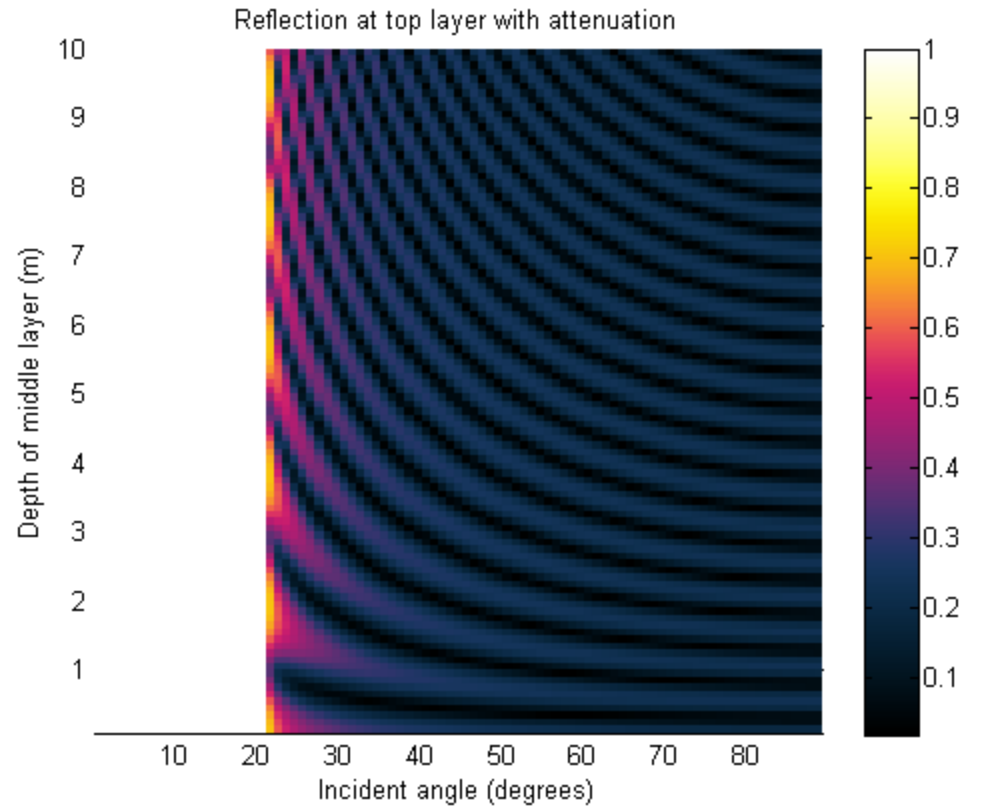
figure('name', 'Reflection at top layer with attenuation');
surf(theta_in_deg, d, reflect', 'EdgeColor', 'none');
colormap(morgensstemming());
colorbar();
view([0 90]);
axis tight;
xlabel('Incident angle (degrees)');
```

```
ylabel('Depth of middle layer (m)');  
title(get(gcf(), 'name'));
```

```
figure('name', 'Transmission at bottom layer with attenuation');  
surf(theta_in_deg, d, transmit', 'EdgeColor', 'none');  
colormap(morgensstemning());  
colorbar();  
view([0 90]);  
axis tight;  
xlabel('Incident angle (degrees)');  
ylabel('Depth of middle layer (m)');  
title(get(gcf(), 'name'));
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate.  
RCOND = 1.951473e-16.  
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RCOND = 1.321916e-16.  
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RCOND = 6.065772e-17.  
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RCOND = 4.108914e-17.  
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RCOND = 2.783350e-17.  
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RCOND = 1.885422e-17.  
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RCOND = 1.192684e-16.  
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RCOND = 8.103333e-17.  
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RCOND = 5.505567e-17.  
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RCOND = 7.869095e-17.
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RCOND = 1.773205e-16.



end

Get the parameters

```
function [R1, T2, R2, T3] = solve_layers(d, theta_in, c, rho)
    theta(1) = theta_in;
    theta(2) = angle_from_snell(theta(1), c(1), c(2));
    theta(3) = angle_from_snell(theta(2), c(2), c(3));

    z(1) = imped(c(1), rho(1), theta(1));
    z(2) = imped(c(2), rho(2), theta(2));
    z(3) = imped(c(3), rho(3), theta(3));

    %k = omega./c;
    k(2) = 2*pi; % 2*pi*d/lambda to non dimensionalize d
    k(3) = k(2)*cos(theta(2))/cos(theta(3));

    p2 = 1i*k(2)*d.*sin(theta(2));
    p3 = 1i*k(3)*d.*sin(theta(3));

    d = [1 1 0 0]';

    A = [-1 1 1 0;
         1 z(1)/z(2) -z(1)/z(2) 0;
         0 exp(p2) exp(-p2) -exp(p3);
         0 exp(p2) -exp(-p2) -(z(2)/z(3))*exp(p3)];

    x = A\d;
    x = abs(x);

    R1 = x(1);
    T2 = x(2);
    R2 = x(3);
    T3 = x(4);
end
```

```
function theta_out = angle_from_snell(theta_in, c1, c2)
    theta_out = acos(cos(theta_in)*c2/c1);
end
```

```
function Z = imped(c, rho, theta)
    Z = rho*c./sin(theta);
end
```

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